Problem 1 [5,10,10, 10 points]: State Charts
Consider a Statecharts description as shown. For this system:

a. Draw the state space of the StateChart as a tree which shows the hierarchy of states and denotes the state types (basic states, sequential and parallel states).

Basic States: \{1, 2, G, D1, D2\}
AND: Parallel or Orthogonal Composition: \( A = \{B, C\} \)
OR: Sequential Composition Composition: \( B = \{1, 2\} \), \( C = \{G, D\} \), \( D = \{D1, D2\} \)

b. How would you formally compute the set of states? Compute the set of states for the hierarchical automata defined by the statechart above. (Hint: Parallel composition of states requires their cross-products.)

“+” represents union, “x” represents cross product

\[
Z_A = Z_B \times Z_C = (Z_1 + Z_2) \times (Z_G + Z_D) \\
= (Z_1 + Z_2) \times (Z_G + (Z_D1 + Z_D2)) \\
= (Z_1, Z_G) + (Z_1, Z_D1) + (Z_1 + Z_D2) + (Z_2, Z_G) + (Z_2, Z_D1) + (Z_2, Z_D2)
\]
c. Consider the external events, a, b, c, d, e as inputs. Draw state transition table showing configurations A, B, C as columns. *For the following sequence of events: a, b, e, b, d, b.* Example:

<table>
<thead>
<tr>
<th>Event</th>
<th>Conf. B</th>
<th>Conf. C</th>
<th>Conf. A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>1</td>
<td>G</td>
<td>1, G</td>
</tr>
<tr>
<td>a</td>
<td>2</td>
<td>D1</td>
<td>2, D1</td>
</tr>
<tr>
<td>b</td>
<td>2</td>
<td>D1</td>
<td>2, D1</td>
</tr>
<tr>
<td>e</td>
<td>2</td>
<td>D2</td>
<td>2, D2</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
<td>D2</td>
<td>1, D2</td>
</tr>
<tr>
<td>d</td>
<td>1</td>
<td>G</td>
<td>1, G</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
<td>G</td>
<td>1, G</td>
</tr>
</tbody>
</table>

This question was ill-posed. Full credit given. Please see the solution above.

d. Draw a finite state machine which is equivalent to the statechart shown. Minimize the number of states. *Six states from (b) but (1,D1) is not reachable.*

![Figure 3: Equivalent FSM (not minimized)](image)

![Figure 4: Equivalent FSM (minimized)](image)
Problem 2 [10,10,20 points]: Synchronous Data Flow

Given the SDF shown below:

(a)

(b)

a. Determine the topological matrix of these two SDF graphs. A topological matrix for q-node SDF $Q=(n_1, n_2, ..., n_q)$ is a qxq matrix such that $MxQ = 0$ defines the flow constraints related to the DF arcs.

\[
\begin{align*}
a - b &= 0 \\
-a + b &= 0 \\
1 &\quad -1 \\
-1 &\quad 1
\end{align*}
\]

\[
\begin{align*}
2a - b &= 0 \\
-a + b &= 0 \\
2 &\quad -1 \\
-1 &\quad 1
\end{align*}
\]

b. Are these two graphs consistent? A graph is consistent if $MxQ = 0$ has a solution. A connected SDF graph with $n$ nodes has a schedule iff its topology matrix $M$ has rank $n-1$.

(a) is consistent, but (b) is not.

c. For the graph below, determine its topology matrix and determine if it is consistent. Find a solution to determine the relative number of node firings for a periodic schedule.
Topological matrix: \[\text{Quelle, DCT, Q, RLC, C, R}\]

\[
\begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & -77 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & -1 & 0 & 0 & 77 \\
\end{bmatrix}
\]

It is consistent. Solution: \([77, 77, 77, 77, 1, 1]\).
Problem 3 [15,10,10 points]: Hybrid Models

a. For the hybrid model of the thermostat discussed in the class and reproduced below, discuss if the model is deterministic or non-deterministic. A deterministic model will have a single trajectory for a given starting condition.

The thermostat regulates temperature, $x$, in the room by switching heater ON and OFF. The thermostat regulates $x$ around 75 degrees by turning ON heater when temperature is between 68 and 70 and turning OFF when the temperature is between 80 and 82.

This model is non-deterministic, since for a given initial condition there is a whole family of state trajectories.

b. Consider the behavior of a bouncing ball shown below:

Here $x_1$ is vertical position of the ball, $x_2$ is the velocity of the ball. $g$ is acceleration due to gravity and $c \in [0, 1]$ is coefficient of restitution. The ball behavior consists of alternating continuous portions (between bounces) and discrete changes at bounce.
times (when $x_1 = 0$). Specifically, when ball is falling downwards, on a bounce it changes its velocity from $x_2$ to $-c \cdot x_2$.

Draw a hybrid state-diagram for the ball that shows the physics and invariant condition for the state(s).

What happens if $c < 1$?

*If c is negative the solution is unbounded increase in x1. The more interesting case is when $c < 1$, the hybrid automaton takes an infinite number of discrete steps in finite time. This behavior is called Zeno, names after Zenos paradox.*